Phase synchronization in temperature and precipitation records

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Abstract

We study phase synchronization between atmospheric variables such as daily mean temperature and daily precipitation records. We find significant phase synchronization between records of Oxford and Vienna as well as between the records of precipitation and temperature in each city. To find the time delay in the synchronization between the records we study the time lag phase synchronization when the records are shifted by a variable time interval of days. We also compare the results of the method with the classical cross-correlation method and find that in certain cases the phase synchronization yields more significant results.
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1. Introduction

In recent years there was much interest in long term persistence of temperature records [1] detected by detrended fluctuation analysis [2]. Fluctuations in space and time of meteorologic records are usually characterized by teleconnection patterns [3]. They describe recurring and persistent patterns of circulation anomalies that take place in huge geographical domains. Prominent patterns are the North Atlantic oscillation (NAO) that appears all over the year, or the East Atlantic pattern (EA), which appears from September to April. Each site in the teleconnection patterns is characterized by

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the strength of the cross-correlation of this site with all other sites in the pattern, for a given meteorologic parameter. By this, a correlation matrix is defined, which usually exhibits two to four regions of extreme high or low values, which are called "centers of action". A more recent measure is the rotated principal component analysis (RCPA), which uses eigenvectors of the correlation (or cross-covariance) matrix after certain scaling and rotation to identify the meteorologic patterns.

The methods to identify teleconnection patterns are based on cross-correlation which essentially compares the amplitude records. Here we suggest an alternative method for studying relations between meteorological records, which is based on the phase synchronization approach [4]. We show that this method can be applied also to complex signals where the fluctuations are not pure oscillations. For certain meteorological records, we find that the phase synchronization approach performs better than the conventional cross-correlation approach. The method also enables to quantify the typical wavelengths of a signal, which cannot be detected by cross-correlation.

The paper is organized as follows: We describe the phase synchronization method in Section 2, present the results in Section 3 and discuss and summarize them in Section 4.

2. Phase synchronization method

The phase synchronization method was originally applied to weakly coupled chaotic oscillators. The method enables to reveal relations between two complex records by focusing on the phases of the fluctuations in each record. The technique was found very useful for identifying phase synchronization in several biological systems, including the synchronization between the breathing cycle and the heart rhythm [5], which reveals the weak interaction between the human respiratory and the cardiovascular system. Analysis of synchronization has also been performed in ecological systems, where complex population oscillations occur [6]. For more applications and a pedagogical review of the method we refer to Ref. [7].

Generally, two periodic oscillators are in resonance, if their frequencies ω_1 and ω_2 are related by

$$n\omega_1 \approx m\omega_2 , \tag{1}$$

where n, m are integers. We define a phase $\phi_j(t) = \omega_j t$ for each oscillator, and the generalized phase-difference is $\varphi_{n,m} = n\phi_1(t) - m\phi_2(t)$. Hence we have resonance for the condition

$$|\varphi_{n,m} - \delta| < \text{const} , \tag{2}$$

where δ represents the phase shift between both oscillators, and the constant on the r.h.s. is any positive finite number. This condition holds also when the frequencies are fluctuating. In this case, $\phi_j(t)$ is calculated for each single record by using a Hilbert transform (see below). In order to test for phase synchronization, we determine [8]

$$\psi_{n,m} = \varphi_{n,m} \bmod 2\pi . \tag{3}$$

If the histogram of $\psi_{n,m}$ shows a maximum at a certain phase-difference, the two records are synchronized at this phase.

In practice, the phase synchronization analysis of two records of length N consists of five steps:

- In the first step, we construct from the scalar signals $\tau_j(t)$, $j = 1, 2$, the complex signals $\zeta_j(t) = \tau_j(t) + i\tau_{H_j}(t) = A_j(t)e^{i\phi_j(t)}$, where $\tau_{H_j}(t)$ is the Hilbert transform of $\tau_j(t)$ [9].
- Then we extract the phases $\phi_1(t)$ and $\phi_2(t)$.
- Next we cumulate the phases such that every cycle, the phases $\phi_j(t)$ increase by 2π.
- Then we quantify the difference of the phases $\varphi_{n,m}(t) = n\phi_1(t) - m\phi_2(t)$.
- Finally, we create a histogram of $\psi_{n,m} = \varphi_{n,m} \mod 2\pi$ for various m and n values. To do this, we subdivide the possible range of the phases $\psi_{n,m}$ into M intervals (bins) of size $2\pi/M$ and determine how often the phase $\psi_{n,m}$ occurs in each interval.

In the absence of phase synchronization, the histogram of $\psi_{n,m}$ is expected to be uniform, because all phase-differences occur with the same probability. In the presence of phase synchronization, there exists, for a certain pair (m, n) a peak in the histogram.

To quantify the significance of synchronization we use an index [8] based on the Shannon entropy S:

$$\rho_{n,m} = \frac{S_{\max} - S}{S_{\max}} \, , \tag{4}$$

where $S = -\sum_{k=1}^{M} p_k \ln p_k$ and p_k is the probability of finding $\psi_{n,m}$ in the kth bin of the histogram. By definition, the maximum entropy is $S_{\max} = \ln M$. The synchronization index is restricted to the unit interval $0 \leqslant \rho_{n,m} \leqslant 1$ and is minimal for a uniform distribution and maximal in the case of a δ-function.

By introducing a time lag into the phase synchronization method, realized by a certain shifting interval S between the two records, it may occur, that for some cases best phase synchronization is found only for a certain time lag. In this case, the synchronization is delayed by this interval, which can be determined by the position of the peak in the synchronization index.

3. Results

We begin the demonstration of the method on the temperature and precipitation records of Oxford (GBR) and Vienna (AUT). The stations are distant enough in order not to give trivial results, but of sufficient closeness for their climate to interact. In order to analyze only the fluctuations, we deseasoned the records by subtracting the annual cycles. We mostly discuss the temperature time series. The values for the first hundred days of the year 1873 for both cities are shown in Fig. 1. Although some similarities can be guessed, the question how to quantify these similarities is of interest. One method
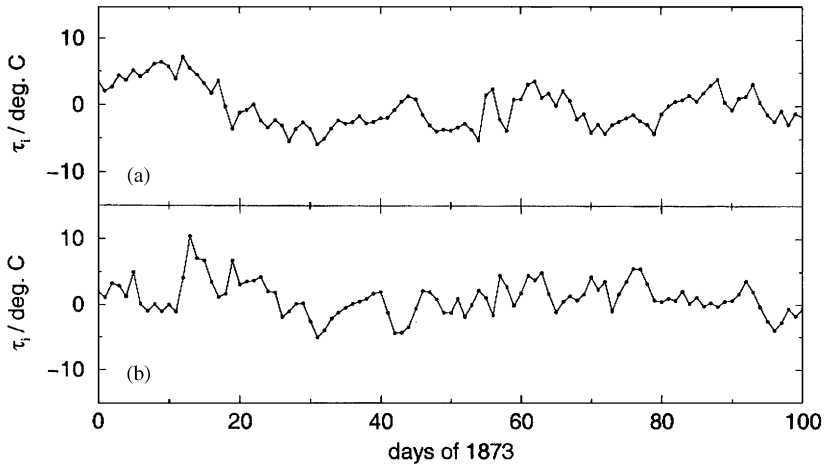
Fig. 1. A typical example of daily mean temperature record for 100 days, starting in 1873 at (a) Oxford (GBR) and (b) Vienna (AUT), after subtraction of the annual cycles, average over all years of the record.

is the cross-correlation approach. Here we propose that complementary information can be revealed by the phase synchronization method.

Figs. 2 and 3 demonstrate the steps of the method. In Fig. 2(a) a small section of the temperature record measured at Oxford is given. The corresponding phases (Fig. 2(b)) were determined using Hilbert transform. Fig. 2(c) shows what the cumulated phases look like, where after every cycle 2π is added. In Fig. 3(a) the cumulated phases for both complete records are shown, while in Fig. 3(b) the phase-differences are displayed. The histogram of the phase-differences modulo 2π, is given in Fig. 3(c). A clear peak can be seen in the histogram and the synchronization index is $\rho = 0.0242$. This value cannot be improved by taking $n{:}m$ other than 1:1.

To get information about the significance of this result we perform time lag phase synchronization, i.e. a shifting-test, where the series are shifted against each other by a given interval of S days. The non-overlapping values in both sequences are ignored for the process. Obviously in the case of no synchronization the value of ρ must be lower than in synchronization. In Fig. 4(a) the result of shifting is shown. For shifting of several days in both directions the synchronization decreases dramatically. A histogram for a shift of +20 days, where $\rho = 0.0011$, is given in Fig. 4(c). This shifting-test reveals, that the non-shifted case does not correspond to the best synchronization. A higher synchronization index ($\rho = 0.0315$) can be achieved with a shift of -1 day. Fig. 4(b) shows this histogram. The peak is slightly sharper and higher than in Fig. 3(c). This result is reasonable since a cycle of fluctuation which is detected in Oxford reaches Vienna (due to high latitude western winds) about 1 day later.

The results are less pronounced for the two precipitation time records. The shifting-test is displayed in Fig. 5(a). Here the importance of the shifting-test becomes clear. Even when shifted, ρ-values of the order of 0.004 are achieved, but the peak is still significantly higher. This high background is probably due to the large fluctuations of
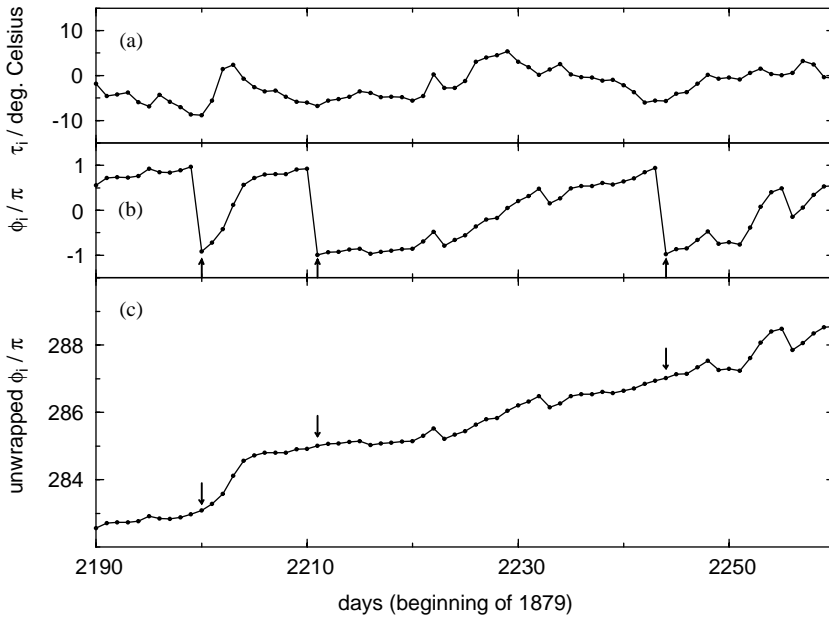
Fig. 2. The steps from the signal to the cumulated phases. (a) Part of the deseasoned temperature record measured at Oxford. (b) The phases extracted by the Hilbert transform. (c) The cumulated phases. The arrows represent the edges of the cycles.
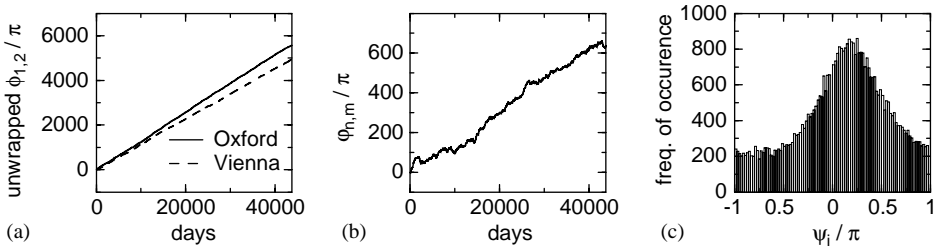

Fig. 3. (a) Cumulated phases for the deseasoned temperature record of Oxford (solid line) and Vienna (dashed line) for the years from 1873 to 1992. (b) Phase-difference. (c) Histogram of phase-difference mod 2π (100 bins).

the precipitation records. They show a spiky structure, that leads the Hilbert transform to give many slips and phases of short duration. Indeed, apart from the long cycles, these records consist of many of 3 to 4-day-periods (shown in Fig. 6(b)). When a pair of precipitation records is shifted, the phases still show matching because of the multitude of very short periods, yielding noise-induced synchronization in the background. Nevertheless a dominant peak is obtained in this representation, the maximum
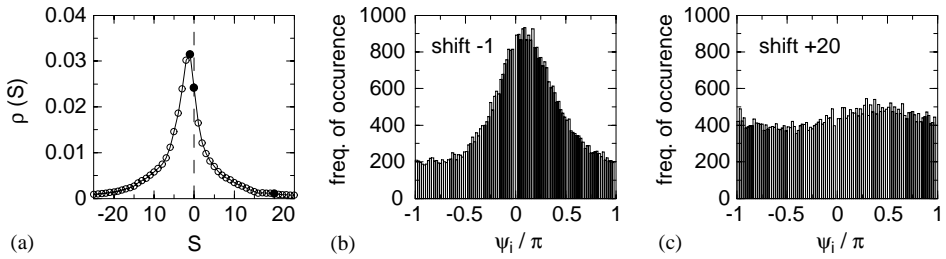
Fig. 4. (a) Dependence of the synchronization index ρ on the shifting interval S between the records of Oxford and Vienna. Negative shifting means that the series of Vienna is in advance. The corresponding non-overlapping values were cut from the records. Histograms for the filled dots are shown in (b) with a shift of -1, (c) with shift of $+20$. Note that in Fig. 3(c) the records are not shifted.
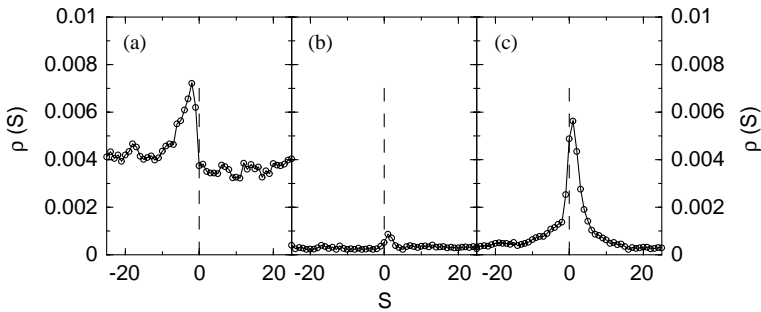

Fig. 5. The synchronization index ρ as a function of the shifting interval S for different pairs of daily records. (a) Precipitation at Oxford and Vienna (1873–1989), whereas for negative shifting values we cut the beginning of Vienna's and the end of Oxford's record. Synchronization between temperature and precipitation measured at (b) Oxford (1873–1992) and (c) Vienna (1873–1989).

synchronization with $\rho = 0.0072$ is reached when the series are shifted by -2 days. Note that this value is a factor 4 smaller than that for temperature.

Synchronization is also found between temperature and precipitation records at the same site. In Oxford the temperature and precipitation records are very weakly synchronized (Fig. 5(b)), with a small peak of $\rho = 0.0009$. At Vienna (Fig. 5(c)) the peak is at least six times larger. In both cases the peaks are located at a time lag of $+1$ day, i.e., they are better synchronized when the temperature record is 1 day in advance to the precipitation record. In comparison to synchronization of the two precipitation time series, they have much less noise-induced synchronization in the background.

The fact that best synchronization for temperature records of Oxford and Vienna is found when they are shifted by one day, exhibits the statistical delay between cycles of weather at both sites. This conclusion is supported by the result for precipitation series, where the maximum ρ occurs for shifting of two days. Probably the real delay is approximately 1.5 days, which, due to low sample-rate, cannot be determined more precisely.
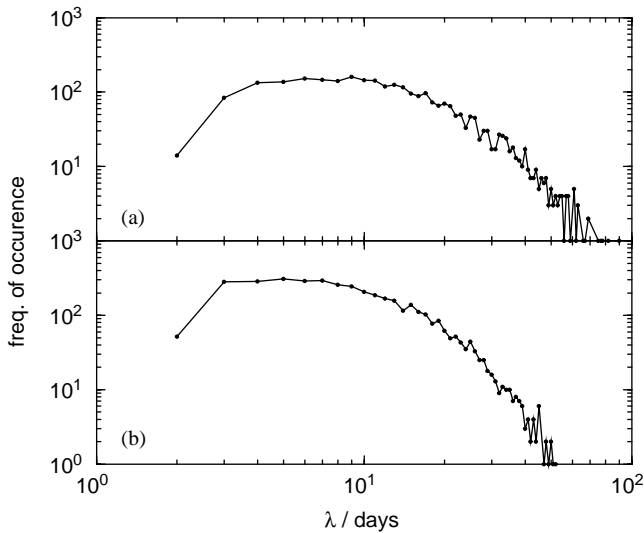
Fig. 6. Histogram of wavelengths for (a) temperature and (b) precipitation records at Oxford, determined by counting the number of days for which the phases complete a cycle.

Usually Fourier transform is applied in order to discover dominant global frequencies or wavelengths in a considered time series. But no direct information can be gained about cycles of varying wavelength, since Fourier transform detects global waves in the record. We suggest to use the cumulated phases to estimate the wavelengths in the time series. Namely, we count the days, until the phases pass steps of 2π, and generate a histogram with frequency of occurrence versus wavelength.

For the temperature and precipitation records of Oxford these histograms are shown in Fig. 6. In the case of temperature (Fig. 6(a)) the fluctuations have a wide range from 2 to about 90 days, but most of them take 5 to 10 days. The precipitation record (Fig. 6(b)) consists of much more fluctuations of short wavelength. The length of 3 days occurs 274 times, while only 84 times in the temperature record. Also the maximum wavelength of the precipitation record is only about 50 days. Note that the considered cycles are not periodic, but have random wavelengths.

4. Discussion

Comparing phase synchronization with the classical cross-correlation method is of interest. While in phase synchronization the phases of the cycles play the major role and not the amplitudes, in cross-correlation both aspects are superimposed. Thus we expect to obtain complementary information from the two approaches. In the following two examples more significant results were obtained from phase synchronization compared to cross-correlation.
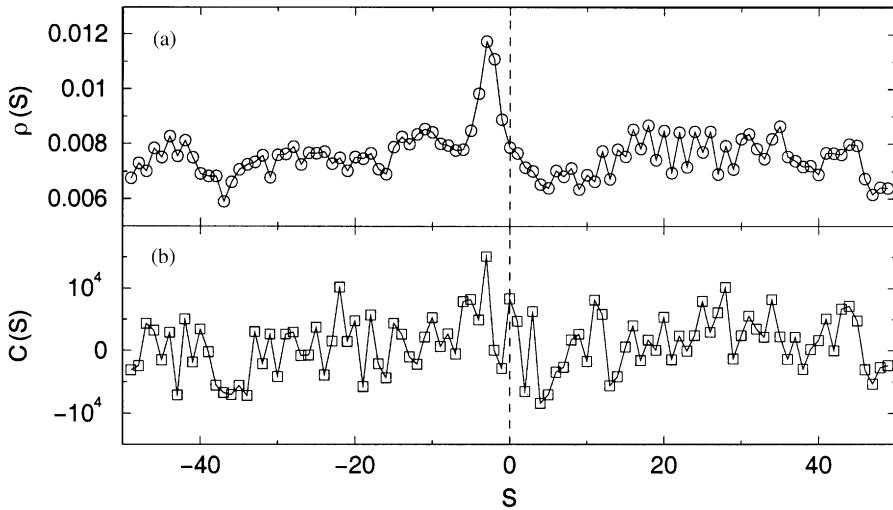
Fig. 7. An example for comparison between (a) phase synchronization and (b) cross-correlation. The methods were applied to the precipitation records of Wulumuqi (CHN) and Pusan (KOR) (daily 1951–1990). Negative shifting means that the dates of Pusan correspond to earlier dates of Wulumuqi.

In Fig. 7 we compare time lag phase synchronization and time lag cross-correlation for precipitation in two sites in Asia. The phase synchronization index exhibits a distinct peak with a maximum at -3 days, while the cross-correlation only gives large background noise with a peak that is almost indistinguishable from the background. Fig. 8 also demonstrates an advantage of phase synchronization. It compares phase synchronization and cross-correlation for records without annual deseasoning. Here only the average value of each record was subtracted. It is seen, that while in the phase synchronization almost a constant background with $\rho \approx 0.19$ is obtained, the cross-correlation shows large annual oscillations, as expected. Thus, the peak in phase synchronization ($\rho = 0.25$) compared to the background (Fig. 8(a)) is much more significant than that in cross-correlation analysis (Fig. 8(b)). The high value of the constant background in the time lag phase synchronization is due to the annual synchronization, which is almost not influenced by variation of the time lag. The peak in the time lag phase synchronization (Fig. 8(a)) is thus mainly due to phase synchronization of the fluctuations which represents irregular cycles of deviation from the mean annual cycle.

Synchronization in the atmosphere plays an important role in climatology. For example tests on an atmospheric global circulation model have been done, where the complete synchronization of the two hemispheres has been analyzed [10]. What does phase synchronization in climate records mean? For temperature records, e.g. a complete relatively warm period followed by a cold period represents a cycle in terms of phases. At another site, which is synchronized to the first, statistically a similar cycle also occurs, maybe with some delay. The amplitudes of these cycles have no influence on the phase synchronization. This is in contrast to cross-correlation, which is strongly affected by the amplitudes. Thus, phase synchronization might be useful
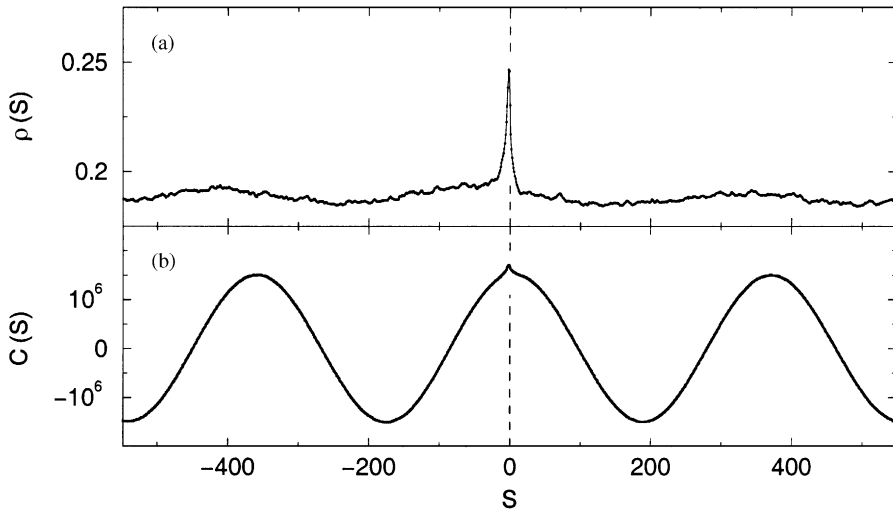
Fig. 8. Comparison of (a) phase synchronization and (b) cross-correlation. Again we analyzed the daily mean temperature records of Oxford and Vienna (1872–1992), but here we subtracted the global average from the record instead of the annual cycle before performing the methods. For positive S the record of Oxford is in advance. Note that (a) corresponds to Fig. 4(a).

when interaction in records of different climate regions is analyzed, such as maritime, where temperature fluctuations are less pronounced, and continental regions with larger fluctuations.

Acknowledgements

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