How to create a ring of N bodies circling around their centre of mass
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For a body to circle around another one the force of gravitation has to be the same as the centripetal force:

$$F = m\frac{v^2}{r} \qquad F = G\frac{m_1 m_2}{r^2} \quad \Rightarrow \quad m\frac{v^2}{r} = G\frac{m_1 m_2}{r^2} \qquad v = \sqrt{G\frac{m_2}{r}} \qquad (*)$$

In the case of a regular ring, the other bodies of the ring have to substitute the central mass. So it is necessary to know a mass m*(m) which fulfils the situation described above in a ring-system.

That means that the force of gravitation effected by the central mass has to be the same as the sum of the parallel (related to the central force) components effected by the ring:

$$F_{CM} = \sum F_n^{\parallel} \qquad (**)$$

Because of the symmetry the perpendicular components to each body are annuled. To determine the parallel components of the forces the distance and angle to each body are needed. Compare the following discussion with the exposed figure:

$$d_1 = 2\cdot r\sin\frac{\alpha}{2} \qquad \beta_1 = \frac{1}{2}\alpha$$

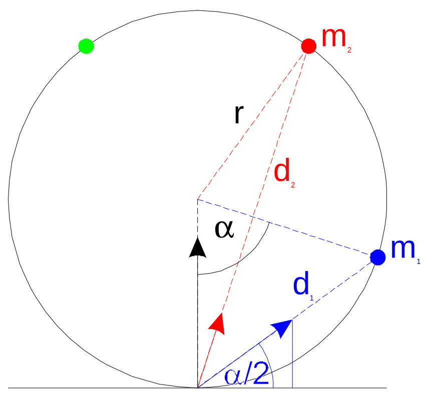
$$d_2 = 2\cdot r\sin\alpha \qquad \beta_2 = \alpha$$

$$d_3 = 2\cdot r\sin\frac{3}{2}\alpha \qquad \beta_3 = \frac{3}{2}\alpha$$

$$\ldots$$

$$d_n = 2\cdot r\sin\frac{n}{2}\alpha \qquad \beta_n = \frac{n}{2}\alpha$$

$$d_n = 2\cdot r\sin\frac{n}{N}\pi \qquad \beta_n = \frac{n}{N}\pi \qquad (\alpha = \frac{360°}{N})$$



The attraction by the nth body can be written as:

$$F_n = G\frac{m_1 m^*}{(2\cdot r\sin\frac{n}{N}\pi)^2}$$

and the parallel component:

$$F_n^{\parallel} = \sin\frac{n}{N}\pi \cdot G\frac{m_1 m^*}{(2\cdot r\sin\frac{n}{N}\pi)^2}$$

To get m* the equation (**) must be solved:

$$G\frac{m_1 m_2}{r^2} = \sum_{n=1}^{N-1} G\frac{m_1 m^*}{4\cdot r^2 \sin\frac{n}{N}\pi}$$

$$m_2 = \frac{m^*}{4}\sum_{n=1}^{N-1}\frac{1}{\sin\frac{n}{N}\pi} \qquad \Rightarrow \qquad \boxed{m^* = \frac{4\cdot m_2}{\sum_{n=1}^{N-1}\frac{1}{\sin\frac{n}{N}\pi}}}$$

or with equation (*):

$$m^* = \frac{4\cdot v^2 r}{G\cdot\sum_{n=1}^{N-1}\frac{1}{\sin\frac{n}{N}\pi}}$$

for example:

N=3: $$\sum_{n=1}^{2}\frac{1}{\sin\frac{n}{N}\pi} = \frac{1}{\sin\frac{\pi}{3}} + \frac{1}{\sin\frac{2}{3}\pi} = \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}} \qquad \Rightarrow \qquad m^* = \sqrt{3}\cdot m_2$$

In the Excel-file „Ring100.xls“ the factor is calculated until N=100.
A linear regression on a logarithmic plot gives a bad approximation:

$$\frac{4}{\sum_{n=1}^{N-1}\frac{1}{\sin\frac{n}{N}\pi}} \approx 10\hat{}\,(-1{,}32033666\cdot \log N + 0{,}74302456)$$

To see how to calculate data for the program GSTIRN-C check the file „16er.xls“ and try „16er.prn“.