On the estimation of damages due to coastal floods

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NG43C: Scaling Functions, Trends, Correlations, and Cross-Correlations in Geosciences and Their Use in Forecasting Natural Hazards I

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Outline

1. Extremes matter
2. Damage functions
3. Expected damages and uncertainty
Sea level rise and fluctuations

 Warnemünde

 meters level

 year


 monthly mean
Sea level rise and fluctuations

[Graph showing fluctuation in sea level from 1996 to 2006 at Warnemünde]

- Y-axis: Level
- X-axis: Year
- The graph shows a trend of fluctuating sea levels with a monthly mean line indicating the average level over time.
Sea level rise and fluctuations

![Graph showing sea level fluctuations over time](image)
Sea level rise and fluctuations
wikipedia: Kupferstich "Deichbruch" von Winterstein 1661
Salacgriva (Latvia)
Motivation

How to estimate damages from (coastal) floods?
How do they change with sea-level-rise?
How are they influenced by protection measures?
Motivation

How to estimate damages from (coastal) floods?

How do they change with sea-level-rise?

How are they influenced by protection measures?

- consider distribution of extremes
- combine with damage function
- study distribution of damages
- dependence on GEV-parameters and protection
Motivation

How to estimate damages from (coastal) floods?

How do they change with sea-level-rise?

How are they influenced by protection measures?
Generalized Extreme Value distributions

The GEV distributions, expressing the probability that the maximum of a sample is beneath the value \( s \), are given by:

\[
P_{(s)}^{\text{GEV}} = \begin{cases} 
\exp \left[ - \left( 1 + \xi \frac{s - \nu}{\gamma} \right)^{-\frac{1}{\xi}} \right] & \text{for } \xi \neq 0 \\
\exp \left[ -e^{-\frac{s - \nu}{\gamma}} \right] & \text{for } \xi = 0.
\end{cases}
\] (1)

They are defined on \( \left\{ s : 1 + \xi \frac{s - \nu}{\gamma} > 0 \right\} \) and have a location parameter, \( \nu \in \mathbb{R} \), a scale parameter, \( \gamma \in \mathbb{R}^+ \), as well as a shape parameter, \( \xi \in \mathbb{R} \). According to the shape, one distinguishes three cases: (i) the Gumbel distribution (\( \xi = 0 \)), (ii) the heavy-tailed Fréchet distribution (\( \xi > 0 \)), and (iii) the bounded-tailed reversed Weibull distribution (\( \xi < 0 \)).
Damage functions

intuitively: the higher the flood, the more damage

damage function: typical damage for flood of certain height

problem: how to determine damage functions?
- empirical data (here: indirectly)
- case study

later: assume power-law
Damage functions from damage records

Which damage function is required so that GEV transforms into observed distribution of damages?

→ density transformation

data: CRED [EM-DAT, 2009], damages due to floods worldwide in the years 1950-2008
Damage functions from damage records

Gumbel: \( D_{(s)} \sim \begin{cases} \frac{e^{-\gamma (\alpha - 1)}}{\gamma s} & \text{for } \tilde{p}(D) \sim D^{-\alpha} \text{ with } \alpha > 1 \\ \left(\frac{1}{\gamma s}\right)^{\frac{1}{\alpha}} & \text{for } \tilde{p}(D) \sim \frac{a}{b} D^{\alpha - 1} e^{-\frac{D^\alpha}{b}} \text{ with } a > 0 \end{cases} \)
Damage functions from damage records

Gumbel:

\[
D_{(s)} \sim \begin{cases} 
    \frac{e^{s(\gamma-1)}}{\gamma} & \text{for } \tilde{p}_D(D) \sim D^{-\alpha} \text{ with } \alpha > 1 \\
    \left(\frac{1}{\gamma} s\right)^{1/a} & \text{for } \tilde{p}_D(D) \sim \frac{a}{b} D^{a-1} e^{-\frac{D^a}{b}} \text{ with } a > 0
\end{cases}
\]

scale parameter

form parameter

Weibull: \( \xi < 0 \)

Gumbel: \( \xi = 0 \)

Frechet: \( \xi > 0 \)

(a) Zipf’s law

(b) stretched exp.
Damage functions from case study - Kalundborg (DK)
Damage functions from case study
- Kalundborg (DK)

hydro-dynamical modeling
more convenient: flood fill
Expected damages and uncertainty

<table>
<thead>
<tr>
<th>damage costs</th>
<th>location $\mu$</th>
<th>scale $\sigma$</th>
<th>protection height $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td></td>
<td></td>
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<tr>
<td>expectation value</td>
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<td>STD</td>
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<tr>
<td>standard deviation</td>
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</tbody>
</table>

- e.g. changing weather patterns
- e.g. sea-level-rise
Expected damages and uncertainty - as a function of the location

damage function: \[ D(x) \sim x^\gamma \]

expectation value: \[ E(C) \sim \mu^\gamma \]

standard deviation: \[ \text{STD}(C) \sim \mu^{\gamma - 1} \]

asymptotically independent from GEV-type (!)
relative uncertainty decreases
damage function exponent is decisive
Expected damages and uncertainty - case study Copenhagen

from S. Hallegatte, Clim. Change, 2011
Expected damages and uncertainty - case study Copenhagen
Expected damages and uncertainty - case study Copenhagen

using local sea level projections from *DIVA*-tool

temporal evolution of expected damage
Expected damages and uncertainty
- as a function of the scale

expectation value:

$$E(C') \sim \sigma^\gamma$$

standard deviation:

$$\text{STD}(C') \sim \sigma^\gamma$$

asymptotically independent from GEV-type (!)
relative uncertainty is constant
damage function exponent is decisive

$$\text{STD}(C') \sim \mu^{\gamma-1}$$
Expected damages and uncertainty - case study Copenhagen
Expected damages and uncertainty - as a function of protection height

I.e. how does the expected annual damage decrease with increasing protection height?

Gumbel: \[ E(C) \sim \omega^\gamma e^{-\omega/\sigma} \] scale parameter

Frechet: \[ E(C) \sim \omega^\gamma \xi^{-1} \] form parameter

Weibull: \[ E(C) \sim (x_{max} - \omega)^{-1/\xi} \] independent of damage function exponent

asymptotically
3 fundamentally different cases
relative uncertainty is increases
Expected damages and uncertainty - case study Copenhagen
Expected damages and uncertainty - case study Kalundborg
Expected damages and uncertainty - overview

<table>
<thead>
<tr>
<th></th>
<th>location $\mu$</th>
<th>scale $\sigma$</th>
<th>protection height $\omega$</th>
</tr>
</thead>
</table>
| E   | $\sim \mu^\gamma$ | $\sim \sigma^\gamma$ | $\sim \omega^\gamma e^{-\omega/\sigma}$ if $\xi = 0$
|     |                |                | $\sim \omega^\gamma^{-1/\xi}$ if $\xi > 0$
|     |                |                | $\sim (x_{max} - \omega)^{-1/\xi}$ if $\xi < 0$
| STD | $\sim \mu^{\gamma-1}$ | $\sim \sigma^\gamma$ | $\sim \omega^\gamma e^{-0.5\omega/\sigma}$ if $\xi = 0$
|     |                |                | $\sim \omega^\gamma^{-0.5/\xi}$ if $\xi > 0$
|     |                |                | $\sim (x_{max} - \omega)^{-0.5/\xi}$ if $\xi < 0$

uncertainty only due to the fact that one does not know when the extremes take place (lower estimate)

differ only by the factor 0.5 in the exponent
Vorstellung einer Bracke oder Durch-Bruch eines Dammes dadurch dass Landt vor jinnen überschwemmet wird, wikipedia, prob. 1718
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Thank you for your attention.

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